

Steel I-Girder Flexure Fundamentals (LRFD Section 6)

Steel I-girders are the most common steel bridge superstructure in the United States, and their flexural design is governed by Article 6.10 of the AASHTO LRFD Bridge Design Specifications and its Appendices A6 and D6. This course walks through the flexural framework: cross-section proportion limits, composite and noncomposite behavior, the compact and noncompact classifications, positive-flexure strength and the ductility check, negative-flexure flange local and lateral-torsional buckling, the web bend-buckling and load-shedding factor, the hybrid factor, constructibility checks during deck placement, and the service and fatigue limits on flange stress. Article and equation numbers follow the 9th Edition (2020); confirm the edition and interims your owner has adopted. Equations were checked against the Caltrans Bridge Design Practice Chapter 6.1 (Steel Design Theory), the Illinois DOT design guide for LRFD composite steel beams, and the FHWA Steel Bridge Design Handbook. The worked numbers in the example sections are illustrative and were computed for this course; they are not from a real bridge.

Learning objectives

- Classify sections: compact and noncompact
- Explain composite and noncomposite behavior
- Check constructibility
- Relate lateral-torsional buckling to bracing

How Article 6.10 is organized

Article 6.10 applies to straight or horizontally curved steel I-section flexural members that are symmetric about the vertical axis in the plane of the web. It is organized by limit state rather than by member type:

- **6.10.1 General:** composite and noncomposite sections, hybrid sections, variable-depth webs, stiffness, flange stresses and moments, minimum negative-flexure deck reinforcement, net-section fracture, web bend-buckling (6.10.1.9) and the web load-shedding (6.10.1.10.2) and hybrid (6.10.1.10.1) factors.
- **6.10.2 Cross-section proportion limits.**
- **6.10.3 Constructibility.**
- **6.10.4 Service limit state.**
- **6.10.5 Fatigue and fracture limit state.**
- **6.10.6 to 6.10.8 Strength limit state:** positive flexure of composite sections in 6.10.7 and discretely braced flanges in compression and tension in 6.10.8.
- **6.10.9 to 6.10.11** cover shear, cover plates and stiffeners, which are outside this course.

Appendix A6 gives an optional, higher flexural resistance for sections in negative flexure (and noncomposite sections) with compact or noncompact webs; Appendix B6 covers moment redistribution; Appendix D6 gives the plastic moment and the depths D_c and D_{cp} .

Proportion limits (6.10.2)

Every I-section must meet the proportion limits before any resistance equation is used. They keep the section in the range for which the resistance equations were developed and help handling and erection.

- **Web without longitudinal stiffeners:** $D/t_w \leq 150$. With longitudinal stiffeners: $D/t_w \leq 300$.
- **Flanges:** $b_f/2t_f \leq 12.0$; $b_f \geq D/6$; $t_f \geq 1.1t_w$; and $0.1 \leq I_{yc}/I_{yt} \leq 10$, where I_{yc} and I_{yt} are the moments of inertia of the compression and tension flanges about the vertical axis.

The $b_f \geq D/6$ rule is often the one that sets the minimum flange width for deep girders, and the I_{yc}/I_{yt} ratio prevents a section so singly symmetric that it behaves like a tee. A narrow top flange that passes these limits may still control during deck placement, which is why constructibility is checked separately.

Composite and noncomposite behavior

Shear connectors make the deck and girder act together. For an unshored girder, the elastic stresses are the sum of stresses from loads applied separately to three sections (6.10.1.1.1a):

- **Steel section alone:** girder self-weight, wet deck concrete and forms, and other permanent loads placed before the deck hardens (DC1).
- **Long-term composite section** (concrete transformed using the modular ratio $3n$): permanent loads applied after the deck hardens, such as barriers and future wearing surface (DC2 and DW). The $3n$ value approximates the effect of creep.
- **Short-term composite section** (modular ratio n): transient loads such as live load plus impact.

In regions of negative flexure, the section used for stress calculations is the steel section plus the longitudinal deck reinforcement within the effective width, unless the concrete is shown not to crack (6.10.1.1.1c). Article 6.10.1.7 requires minimum longitudinal reinforcement of 1 percent of the deck cross-sectional area where the tensile stress in the deck under the specified loads exceeds ϕf_r , with two-thirds placed in the top layer, to control cracking.

For strength in positive flexure, a composite compact section is designed to its plastic moment, computed by Appendix D6 with the concrete at $0.85f'_c$ and the steel at yield. That is why the order of loading does not matter for the compact positive-flexure strength check but does matter for elastic checks such as service and constructibility.

Compact, noncompact and slender: what the words mean

Steel flexural members are classified by how far they can go before a local or global instability stops them. A **compact** section can reach its full plastic moment M_p and sustain some rotation. A **noncompact** section can reach at least first yield but buckles locally before full plastification. A **slender** element buckles elastically before yield. On a moment-curvature plot, the compact section rises to M_p and flattens out; the noncompact section peaks between M_y and M_p ; the slender section falls off below M_y .

Article 6.10 applies this idea in two different ways depending on the region:

- **Composite sections in positive flexure** are classified as a whole, as compact or noncompact, because the deck braces the compression flange continuously and the web is mostly in tension.
- **Sections in negative flexure, and noncomposite sections,** are checked flange by flange. The compression flange stress is limited by flange local buckling and lateral-torsional buckling, with the web's slenderness entering through the load-shedding factor R_b (6.10.8). Appendix A6 is an optional route that lets sections with compact or noncompact webs develop moments above M_y .

Classifying a composite section in positive flexure (6.10.6.2.2)

A composite section in positive flexure in a straight bridge qualifies as compact when all of the following are met:

- The specified minimum yield strengths of the flanges do not exceed 70 ksi.
- The web satisfies the 6.10.2.1.1 limit, $D/t_w \leq 150$, so no longitudinal stiffeners are required.
- The web satisfies $2D_{cp}/t_w \leq 3.76\sqrt{(E/F_{yc})}$, where D_{cp} is the depth of web in compression at the plastic moment, computed by Appendix D6.3.2.

For most composite girders the plastic neutral axis lies in the top flange or very high in the web, so D_{cp} is zero or small and the web limit is easily met. A section that fails any condition is noncompact and is designed by 6.10.7.2, which limits flange stresses: the compression flange to $\phi_f R_b R_h F_{yc}$ and the tension flange to $\phi_f R_h F_{yt}$, computed elastically from the loads applied to the appropriate sections. Horizontally curved bridges are treated as noncompact in positive flexure under this article.

Compact positive-flexure resistance (6.10.7.1)

For a compact composite section in positive flexure, the strength check is $M_u + (1/3)f_l S_{xt} \leq \phi_f M_n$ (6.10.7.1.1-1), where f_l is the flange lateral bending stress in the tension flange and $S_{xt} = M_{yt}/F_{yt}$. In a straight girder without significant wind or skew effects, f_l is often small. The resistance factor ϕ_f is 1.0 (6.5.4.2).

The nominal resistance depends on the depth of the plastic neutral axis below the top of the deck, D_p , compared with the total depth of the composite section, D_t :

- If $D_p \leq 0.1D_t$: $M_n = M_p$ (6.10.7.1.2-1).
- Otherwise: $M_n = M_p(1.07 - 0.7D_p/D_t)$ (6.10.7.1.2-2).

The reduction reflects that a deep plastic neutral axis means large compressive strains in the deck before the steel fully yields, so the concrete may crush before M_p is reached. At $D_p/D_t = 0.1$ the second equation gives exactly M_p , so the two meet without a jump.

In a continuous span, M_n is further limited to $1.3R_h M_y$ (6.10.7.1.2-3) unless the span and the adjacent pier sections meet conditions given in that article. The cap protects against positive-moment sections shedding more moment to the piers than the pier sections can take if they were designed elastically.

The ductility check (6.10.7.3)

Independent of whether the section is compact, every composite section in positive flexure must satisfy $D_p \leq 0.42D_t$ (6.10.7.3-1). The Illinois DOT guide notes that this check is analogous to the tension-controlled limit for reinforced concrete: it keeps the tension flange yielding well before the deck crushes, so the girder fails in a ductile way. A section that fails the ductility check usually has a deck that is thin relative to a heavy bottom flange, and the remedy is to resize the steel or deck rather than to change the resistance equation.

Note that D_p is measured from the top of the concrete deck, so the haunch and deck thickness are included. Designers sometimes neglect the haunch to be conservative for strength, but for D_p and D_t consistency matters more: use the same assumptions in both.

Discretely braced compression flanges (6.10.8.1 and 6.10.8.2)

In negative flexure at a pier, and in any noncomposite girder, the compression flange is braced only at cross-frames or diaphragms. Article 6.10.8.1.1 requires $f_{bu} + (1/3)f_l \leq \phi F_{nc}$, where f_{bu} is the flange major-axis stress without lateral bending and f_l is the lateral bending stress. The nominal flexural resistance of the compression flange F_{nc} is the smaller of the flange local buckling and lateral-torsional buckling resistances (6.10.8.2.1).

Flange local buckling (6.10.8.2.2)

With $\lambda_f = b_{fc}/2t_{fc}$, the compact limit $\lambda_{pf} = 0.38\sqrt{E/F_{yc}}$ and the noncompact limit $\lambda_{rf} = 0.56\sqrt{E/F_{yr}}$:

- If $\lambda_f \leq \lambda_{pf}$: $F_{nc} = R_b R_h F_{yc}$.
- Otherwise: $F_{nc} = [1 - (1 - F_{yr}/R_h F_{yc})(\lambda_f - \lambda_{pf})/(\lambda_{rf} - \lambda_{pf})] R_b R_h F_{yc}$.

F_{yr} is the compression flange stress at the onset of nominal yielding including residual stress effects, taken as the smaller of $0.7F_{yc}$ and F_{yw} , but not less than $0.5F_{yc}$. For Grade 50 steel, λ_{pf} is about 9.2. Since 6.10.2.2 already limits $b_f/2t_f$ to 12, the flange is never slender in the elastic range, which is why there is no elastic flange local buckling equation.

Lateral-torsional buckling (6.10.8.2.3)

The limiting unbraced lengths are $L_p = 1.0r_t\sqrt{E/F_{yc}}$ and $L_r = \pi r_t\sqrt{E/F_{yr}}$, where r_t is the effective radius of gyration for lateral-torsional buckling: $r_t = b_{fc}/\sqrt{[12(1 + D_c t_w/3b_{fc} t_{fc})]}$. It is the radius of gyration of the compression flange plus one-third of the web in compression.

- If $L_b \leq L_p$: $F_{nc} = R_b R_h F_{yc}$.
- If $L_p < L_b \leq L_r$ (inelastic): $F_{nc} = C_b [1 - (1 - F_{yr}/R_h F_{yc})(L_b - L_p)/(L_r - L_p)] R_b R_h F_{yc} \leq R_b R_h F_{yc}$.
- If $L_b > L_r$ (elastic): $F_{nc} = F_{cr} \leq R_b R_h F_{yc}$, with $F_{cr} = C_b R_b \pi^2 E / (L_b/r_t)^2$.

The moment gradient modifier C_b

C_b accounts for the benefit of a moment that varies along the unbraced length: a flange that is highly stressed at only one end of the segment is less prone to buckle than one under uniform stress. For unbraced cantilevers, members where $f_{mid}/f_2 > 1$, or where $f_2 = 0$, $C_b = 1.0$. Otherwise $C_b = 1.75 - 1.05(f_1/f_2) + 0.3(f_1/f_2)^2 \leq 2.3$ (6.10.8.2.3-7). Here f_2 is generally the largest compressive stress at either end of the unbraced length, f_{mid} is the stress at the middle, and f_1 is defined in that article from f_0 , f_{mid} and f_2 so that it represents the stress at the other end of a linear stress gradient. Stresses for C_b are taken as positive in compression.

Two practical points follow. First, C_b only helps in the inelastic and elastic ranges; it never raises F_{nc} above $R_b R_h F_{yc}$. Second, for the wet-concrete constructibility check of a simple span, the moment near midspan is nearly uniform within one cross-frame bay, so C_b near 1.0 is the realistic value there. Designers who assume a larger C_b without computing it are making an unconservative shortcut.

Relating LTB to bracing layout

The LTB equations make the link between resistance and bracing explicit. Shortening L_b by adding cross-frames, widening the compression flange to increase r_t , or reducing $D_c t_w$ all raise F_{nc} . Each has a cost. Cross-frames add fabrication and erection cost and fatigue-sensitive connection details; wider flanges add steel weight. The FHWA Steel Bridge Design Handbook discusses this trade-off in its volumes on bracing and on constructibility.

Article 6.7.4 governs the cross-frames and diaphragms themselves. They must be designed for the forces they carry (for example, wind, construction loads and the effects of skew or curvature), and in straight, non-skewed bridges their spacing is often set by the constructibility check of the top flange during deck placement and by the negative-moment bottom flange near piers. Because the bottom flange at a pier is in compression, the cross-frames there brace it; the top flange in positive regions is continuously braced by the hardened deck but not before.

For tension flanges that are discretely braced, 6.10.8.3 limits $f_{bu} + (1/3)f_l \leq \phi_t F_{nt}$, with $F_{nt} = R_h F_{yt}$. Tension flanges do not buckle, so only yielding is checked.

Web bend-buckling (6.10.1.9)

A tall, thin web in flexural compression can buckle in a pattern of bulges along its compression zone, known as bend-buckling. Unlike flange buckling, web bend-buckling does not end the girder's usefulness, because the web has post-buckling strength and sheds compressive stress to the flange. AASHTO therefore treats it differently depending on the limit state.

For webs without longitudinal stiffeners, the nominal bend-buckling resistance is $F_{crw} = 0.9Ek/(D/t_w)^2$ (6.10.1.9.1-1), limited to the smaller of $R_h F_{yc}$ and $F_{yw}/0.7$. The bend-buckling coefficient is $k = 9/(D_c/D)^2$. A web with half its depth in compression ($D_c/D = 0.5$) has $k = 36$; a web with more of its depth in compression has a lower k and buckles sooner. The Caltrans and course-note summaries of this article both emphasize that D_c is the elastic depth of web in compression, computed from the summed stresses on the steel, long-term and short-term sections for composite girders (Appendix D6.3.1).

F_{crw} appears directly in three places: the constructibility check of the noncomposite girder (6.10.3.2.1), the Service II check of web compression (6.10.4.2.2), and the special fatigue check for webs in older editions. At the strength limit state the web's slenderness enters through R_b instead.

The web load-shedding factor R_b (6.10.1.10.2)

When a slender web bend-buckles at the strength limit state, part of the web's compressive stress is shed to the compression flange, which then reaches its limit at a lower applied moment. R_b reduces the flange resistance to reflect this:

- $R_b = 1.0$ when checking constructibility, for composite sections in positive flexure that meet the web limit $D/t_w \leq 150$, for sections with longitudinal stiffeners meeting a stated limit, or when $2D_c/t_w \leq \lambda_{rw}$.
- Otherwise $R_b = 1 - [a_{wc}/(1200 + 300a_{wc})](2D_c/t_w - \lambda_{rw}) \leq 1.0$ (6.10.1.10.2-3).

Here $\lambda_{rw} = 5.7\sqrt{(E/F_{yc})}$, which is about 137 for Grade 50 steel, and $a_{wc} = 2D_c t_w / (b_{fc} t_{fc})$, the ratio of twice the web area in compression to the compression flange area. The equation shows that a heavier compression flange (small a_{wc}) is less affected by web load shedding, which is intuitive: a big flange can absorb the shed stress.

Because R_b applies at strength but is taken as 1.0 for constructibility, the constructibility check uses F_{crw} directly to keep the web from bend-buckling while the girder is noncomposite. Web bend-buckling during construction is undesirable because it can introduce out-of-plane distortion that becomes permanent.

Hybrid sections and the hybrid factor R_h (6.10.1.10.1)

A hybrid girder uses a lower-strength web with higher-strength flanges, for example a Grade 50 web with HPS Grade 70W flanges. Under moment, the web near the flanges yields before the flanges do, so the section reaches its nominal flange stress later than a homogeneous section would predict. R_h accounts for this reduction.

For homogeneous sections $R_h = 1.0$. For hybrid sections, $R_h = (12 + \beta(3\rho - \rho^3))/(12 + 2\beta)$ (6.10.1.10.1-1), where $\beta = 2D_n t_w / A_{fn}$ and ρ is the smaller of F_{yw}/f_n and 1.0. D_n is the larger of the distances from the elastic neutral axis to the inside faces of the flanges, A_{fn} is the flange area on the side of the neutral axis corresponding to D_n (including any cover plate and, for composite sections in negative flexure, the deck reinforcement as specified), and f_n is the corresponding yield stress or first-yield stress. R_h is usually only a little below 1.0 for practical hybrid girders, but it multiplies every flange resistance in Article 6.10 and the Service II limits.

Article 6.10.1.3 sets limits on hybrid sections, including a minimum web yield strength relative to the flanges. Check the adopted edition for the current limits, because the hybrid provisions have been revised across editions as HPS steels became common.

Constructibility (6.10.3)

A composite girder spends its most vulnerable stage as a bare steel section. During deck placement the top flange is in compression, braced only at cross-frames, and carries the full weight of wet concrete, forms, screed and workers. Article 6.10.3 requires the girder to be checked for each critical construction stage, using the factored loads applied to the noncomposite section and any section made composite by an earlier pour.

Discretely braced flanges in compression (6.10.3.2.1)

For critical stages of construction, the compression flange must satisfy:

- $f_{bu} + f_l \leq \phi_f R_h F_{yc}$ (6.10.3.2.1-1), a yielding check including lateral bending;
- $f_{bu} + (1/3)f_l \leq \phi_f F_{nc}$ (6.10.3.2.1-2), the flange local and lateral-torsional buckling check, with F_{nc} computed from 6.10.8.2 but with R_b taken as 1.0; and
- $f_{bu} \leq \phi_f F_{crw}$ (6.10.3.2.1-3), the web bend-buckling check.

Two exceptions apply, as summarized in Caltrans BDP 6.1: for sections with compact or noncompact webs, Eq. 6.10.3.2.1-3 is not checked, and for sections with slender webs, Eq. 6.10.3.2.1-1 is not checked when f_l is zero. The web must also satisfy $V_u \leq \phi_v V_{cr}$ under the factored construction shears (6.10.3.3). Discretely braced flanges in tension must satisfy $f_{bu} + f_l \leq \phi_f R_h F_{yt}$. Continuously braced flanges in compression or tension (for example, a flange encased in a previously cured deck pour) need only $f_{bu} \leq \phi_f R_h F_{yf}$, plus the web check for compression.

Flange lateral bending

In constructibility, the lateral bending stress f_l is often significant in exterior girders because deck overhang brackets push on the web and the overhang weight is eccentric to the girder. Article 6.10.1.6 limits f_l to $0.6F_{yf}$. It allows a first-order f_l when the unbraced length satisfies $L_b \leq 1.2L_p \sqrt{(C_b R_b / (f_{bu} / F_{yc}))}$; otherwise the first-order value is amplified by $0.85 / (1 - f_{bu} / F_{cr})$, not less than the first-order value, to account for second-order effects. A common approximation for the first-order lateral moment from a uniform lateral load between cross-frames is the fixed-end value $F_l L_b^2 / 12$; Article C6.10.3.4 discusses overhang effects.

Deck placement sequence (6.10.3.4)

In continuous spans, the deck is often placed in a specified sequence, and the moments in the girders at each stage depend on which portions have already hardened. 6.10.3.4 requires that sections be checked for the sequence; the maximum positive moment during placement may exceed the moment from placing the whole deck at once. Article 6.10.3.2.4 also limits the tensile stress in previously placed concrete deck during later pours, with longitudinal reinforcement required where the limit is exceeded. Contract plans should show the assumed sequence so the contractor knows the design basis.

Load factors

Article 3.4.2.1 sets minimum load factors for construction-stage strength checks, including a minimum on DC and DW and a higher factor for construction loads and equipment. Use the values in the adopted edition and any owner requirements.

Service limit state: permanent deformation (6.10.4)

The Service II load combination (live load factor 1.3, Table 3.4.1-1) represents heavy traffic that may cross occasionally and is used to keep steel structures from permanent deformation that would hurt rideability. Article 6.10.4.2.2 limits flange stresses calculated elastically from Service II loads applied to the appropriate sections:

- Top steel flange of composite sections: $f_f \leq 0.95R_h F_{yf}$.
- Bottom steel flange of composite sections: $f_f + f_l / 2 \leq 0.95R_h F_{yf}$.
- Both flanges of noncomposite sections: $f_f + f_l / 2 \leq 0.80R_h F_{yf}$.

Also under Service II, for sections other than composite sections in positive flexure that satisfy $D/t_w \leq 150$ (with no longitudinal stiffeners), the compression-flange stress must not exceed F_{crw} , so the web does not bend-buckle at service. For composite sections in negative flexure, the specification allows the concrete deck to be assumed effective for Service II stress calculations if certain conditions about minimum reinforcement and deck tensile stress are met; this matters because including the deck reduces calculated flange stresses.

Live load deflection limits in 2.5.2.6.2 are optional unless the owner invokes them. The commonly quoted values are span/800 for vehicular load and span/1000 for vehicular and pedestrian load in urban areas. Many owners adopt them.

Fatigue and fracture (6.10.5)

Article 6.10.5 sends the designer to 6.6.1 for load-induced fatigue of details, such as welded cross-frame connection plates, stud shear connectors on flanges, flange splices and the web-to-flange welds. Each detail has a category (A through E') in Table 6.6.1.2.3-1 with a constant-amplitude fatigue threshold and a finite-life resistance curve. The Fatigue I combination

(load factor 1.75 in the 9th Edition) checks infinite life; Fatigue II (0.80) checks finite life where the projected truck traffic is below the infinite-life threshold.

For flexural design, the main point is that fatigue often controls the location and type of attachments rather than flange size. A Category E' cover plate end, for example, has a very low threshold and is now rarely used for that reason. Cross-frame connection plates welded to the tension flange are Category C' under the stated conditions, which is why many owners now require them to be welded to both flanges, avoiding the distortion-induced cracking seen in older bridges where the plates were cut short of the tension flange. Fracture toughness requirements (6.6.2) for members in tension, including fracture-critical members, are specified through Charpy V-notch values on the plans.

Article 6.10.5.3 adds a special fatigue requirement for webs in shear, which is covered with the shear provisions rather than here. Fatigue of the deck and shear connectors is part of composite design and is handled in 6.10.10.

A note on C_b from Caltrans practice

Caltrans Bridge Design Practice Chapter 6.1 points out a limitation that is easy to miss: the AASHTO C_b equation has been in the LRFD specifications since 1994 and applies to *linearly varying* moment between brace points, which is rare in bridge girders. It can be misinterpreted when the moment diagram is curved within the unbraced segment. For that reason Caltrans amendments adopt the Kirby and Nethercot form also used by AISC: $C_b = 12.5M_{\max}/(2.5M_{\max} + 3M_A + 4M_B + 3M_C)$, with M_A , M_B and M_C the absolute moments at the quarter, center and three-quarter points of the segment, and $C_b = 1.0$ for cantilevers with an unbraced free end. If you work for a Caltrans project, use the California amendments; elsewhere, use the national equation as written, defining f_1 and f_2 exactly as the specification does.

Illustrative example, part 1: proportions and the plastic moment

All numbers in this example are illustrative, chosen and computed for this course. They do not come from a real bridge, and loads and dimensions were not designed for any specific project.

Section. Grade 50 homogeneous plate girder ($F_y = 50$ ksi, $E = 29,000$ ksi, $R_h = 1.0$): top flange 16×1 in.; web $54 \times 9/16$ in.; bottom flange $18 \times 1\frac{1}{2}$ in. Composite deck 8 in. thick, effective width 96 in., $f'_c = 4$ ksi, 2-in. haunch from the bottom of deck to the top of the top flange. Deck reinforcement is neglected in the plastic moment for simplicity. Simple span, so the $1.3R_hM_y$ cap for continuous spans does not apply.

Proportion limits (6.10.2). $D/t_w = 54/0.5625 = 96 \leq 150$. Top flange $b/t = 8.0$ and bottom 6.0 , both ≤ 12 . $D/6 = 9$ in., and both flanges are wider. $t_f \geq 1.1t_w = 0.62$ in. is met. $I_{yc}/I_{yt} = (1 \times 16^3)/(1.5 \times 18^3) = 4,096/8,748 = 0.47$, between 0.1 and 10. All pass.

Plastic forces (Appendix D6.1). Deck $P_s = 0.85(4)(96)(8) = 2,611$ kips; top flange $P_c = 50(16)(1) = 800$ kips; web $P_w = 50(54)(0.5625) = 1,519$ kips; bottom flange $P_t = 50(18)(1.5) = 1,350$ kips.

Locate the PNA. $P_t + P_w + P_c = 3,669$ kips exceeds P_s , so the PNA is in the steel. $P_t + P_w = 2,869$ kips is less than $P_c + P_s = 3,411$ kips, so it is in the top flange, at $\bar{y} = (t_c/2)[(P_w + P_t - P_s)/P_c + 1] = 0.5[(2,869 - 2,611)/800 + 1] = 0.66$ in. below the top of the top flange.

Plastic moment. With distances from the PNA to each force: deck $4 + 2 + 0.66 = 6.66$ in.; web $0.34 + 27 = 27.34$ in.; bottom flange $0.34 + 54 + 0.75 = 55.09$ in. $M_p = (P_c/2t_c)[\bar{y}^2 + (t_c - \bar{y})^2] + P_s d_s + P_w d_w + P_t d_t \approx 221 + 17,390 + 41,520 + 74,370 \approx 153,500$ kip-in. $\approx 12,790$ kip-ft.

Compactness and resistance. The PNA is in the flange, so $D_{cp} = 0$ and $2D_{cp}/t_w = 0 \leq 3.76\sqrt{(E/F_{yc})} = 90.6$. With $F_y \leq 70$ ksi and $D/t_w \leq 150$, the section is compact. $D_p = 8 + 2 + 0.66 = 10.66$ in. and $D_t = 8 + 2 + 1 + 54 + 1.5 = 66.5$ in. Since $D_p > 0.1D_t = 6.65$ in., $M_n = M_p(1.07 - 0.7 \times 10.66/66.5) = 0.958M_p \approx 12,250$ kip-ft. Ductility: $D_p = 10.66 \leq 0.42D_t = 27.9$ in. OK. With $\phi_f = 1.0$, the factored positive moment (plus the f_l term) must not exceed about 12,250 kip-ft.

Illustrative example, part 2: the bare steel girder during deck placement

Illustrative numbers again, continuing the same section.

Noncomposite properties. Steel area 73.4 in.². Elastic neutral axis 24.29 in. above the bottom of the bottom flange; $I_x \approx 38,980$ in.⁴; section modulus to the top flange $S_t \approx 1,210$ in.³. Depth of web in compression $D_c = 56.5 - 24.29 - 1.0 = 31.21$ in.

Effective radius of gyration. $r_t = 16/\sqrt{[12(1 + 31.21 \times 0.5625/(3 \times 16 \times 1))]} = 3.95$ in.

Limiting lengths. $F_{yr} = \min(0.7 \times 50, 50) = 35$ ksi ($\geq 0.5F_{yc}$). $L_p = 1.0(3.95)\sqrt{(29,000/50)} = 95.2$ in. = 7.9 ft. $L_r = \pi(3.95)\sqrt{(29,000/35)} = 357$ in. = 29.8 ft.

Flange local buckling. $\lambda_f = 8.0 \leq \lambda_{pf} = 0.38\sqrt{(29,000/50)} = 9.15$, so $F_{nc(FLB)} = R_b R_h F_{yc} = 50$ ksi ($R_b = 1.0$ for constructibility).

Lateral-torsional buckling, $C_b = 1.0$ conservatively.

Cross-frame spacing L_b	Range	$F_{nc(LTB)}$
20 ft	Inelastic	$[1 - 0.3(240 - 95.2)/(357 - 95.2)](50) \approx 41.7$ ksi
25 ft	Inelastic	≈ 38.3 ksi
30 ft	Elastic (just over L_r)	$F_{cr} = \pi^2(29,000)/(360/3.95)^2 \approx 34.5$ ksi

Web bend-buckling. $k = 9/(31.21/54)^2 = 26.9$; $0.9Ek/(D/t_w)^2 = 0.9(29,000)(26.9)/96^2 \approx 76$ ksi, which exceeds $R_h F_{yc}$, so $F_{crw} = 50$ ksi. Also $2D_c/t_w = 111 < \lambda_{rw} = 137$, so this web is not slender and, per the exception noted earlier, Eq. 6.10.3.2.1-3 would not govern anyway.

Applying it. Suppose (illustratively) a 100-ft simple span with cross-frames at 25 ft, and the wet deck, haunch, forms and girder weight total 1.2 kips/ft per girder before factoring. Unfactored $M = 1.2(100)^2/8 = 1,500$ kip-ft. Using an assumed construction load factor of 1.25 for this illustration (check 3.4.2.1 and the owner for the governing factors and any construction live load), $M_u \approx 1,875$ kip-ft and $f_{bu} = 1,875 \times 12/1,210 \approx 18.6$ ksi. Against $\phi_f F_{nc} = 38.3$ ksi there is a comfortable margin if f_l is small, but an exterior girder with overhang bracket loads could have a significant f_l , which enters fully in Eq. 6.10.3.2.1-1 and by one-third in Eq. 6.10.3.2.1-2.

What the example shows. Moving from 25-ft to 30-ft cross-frame spacing drops the LTB resistance about 10 percent here, while a 2-in. wider top flange would raise r_t and L_r . The bare-girder check, not the composite strength check, is what makes the top flange and bracing matter in positive moment regions: the composite section in part 1 had a large reserve, while the noncomposite girder depends entirely on its bracing.

A flexural design workflow

The provisions above fit together in a fairly standard order. The sequence below is a study aid, not a substitute for the specification or an owner's design manual.

1. **Trial section.** Pick web depth from span-to-depth guidance and owner preference, then flange sizes. Check the 6.10.2 proportion limits immediately; they are cheap to check and save rework.
2. **Section properties.** Compute the steel, 3n and n section properties for positive regions and the steel-plus-reinforcement section for negative regions. Compute D_c from summed stresses and, for strength, D_{cp} and M_p from Appendix D6.
3. **Constructibility (6.10.3).** Check each deck placement stage on the noncomposite section with the governing construction load factors. Include overhang bracket lateral bending in exterior girders. This step often fixes the top flange width and cross-frame spacing.
4. **Service II (6.10.4).** Check flange stresses against $0.95R_h F_{yf}$ (composite) or $0.80R_h F_{yf}$ (noncomposite), and web compression against F_{crw} where required. Check live load deflection if the owner requires it.
5. **Fatigue (6.10.5 and 6.6.1).** Check stress ranges at each detail: connection plates, studs, splices and any welded attachments.
6. **Strength (6.10.6 to 6.10.8).** In positive flexure, classify compact or noncompact, compute M_n and check ductility. In negative flexure, compute F_{nc} from flange local and lateral-torsional buckling with R_b and R_h , and check the tension flange. Consider Appendix A6 where the web is compact or noncompact and the extra resistance is worthwhile.
7. **Iterate.** Adjust flange plates, transition locations and bracing, and repeat. Flange transitions are often placed at field splices or where the savings in steel justify a butt weld.

Common mistakes

- **Using the composite section for wet-concrete stresses.** The deck is not composite until it hardens. For unshored construction, wet concrete goes on the steel section alone.
- **Mixing up D_c and D_{cp} .** D_c is elastic and used for web bend-buckling, R_b and r_t ; D_{cp} is plastic and used for the compact web limit. For a composite positive section, D_c changes with the load stage, so use the summed-stress definition.
- **Assuming C_b without computing it.** Near midspan, the moment within one bay is nearly uniform, so C_b is close to 1.0. Values above 1.0 must be justified with the specification's definitions of f_1 and f_2 .
- **Ignoring flange lateral bending.** Overhang brackets, skew, wind and curvature all create f_1 . It is small in many interior girders but can govern exterior girders during construction.
- **Forgetting the ductility check.** A section can be compact yet fail $D_p \leq 0.42D_t$. The two checks are independent.
- **Applying the $1.3R_h M_y$ cap in a simple span, or missing it in a continuous one.** It applies to continuous spans unless the article's conditions are met.

- **Using outdated article numbers or equations.** Section 6 was reorganized substantially in the 2004–2005 era and has been revised since. Confirm the edition and interim revisions the owner has adopted, and use the owner's amendments, such as California's C_b equation.

Summary and self-check

Steel I-girder flexure in LRFD is a chain of checks on one section through its life: as bare steel during deck placement, as a composite section under service and fatigue loads, and at strength, where the positive region is checked on a plastic or compact basis and the negative region on a flange-stress basis that accounts for local and lateral-torsional buckling. The proportion limits keep the section within the range of the equations; R_b and R_h carry web slenderness and hybrid effects into the flange resistances; and the bracing layout enters through L_b , r_t and C_b .

Self-check questions (answers follow):

1. A Grade 50 composite girder in positive flexure has its PNA in the web with $2D_{cp}/t_w = 95$. Is it compact?
2. For the bare girder in the illustrative example, what is the lateral-torsional buckling resistance if cross-frames are at 7 ft?
3. Why is R_b taken as 1.0 for a composite section in positive flexure with $D/t_w \leq 150$?
4. Which section is used to compute stresses from the future wearing surface on an unshored composite girder?

Answers. (1) No: 95 exceeds $3.76\sqrt{(29,000/50)} \approx 90.6$, so it is noncompact and designed by 6.10.7.2. (2) 7 ft is less than $L_p \approx 7.9$ ft, so $F_{nc} = R_b R_h F_{yc} = 50$ ksi. (3) The deck carries most of the compression and the web is mostly in tension, so the web is not prone to bend-buckling at strength. (4) The long-term composite section, using the modular ratio $3n$.

Where Appendix A6 fits

The flange-stress approach of Article 6.10.8 caps the compression flange at $R_b R_h F_{yc}$, which corresponds roughly to first yield of the flange. For sections whose webs are stocky enough, the girder can actually carry more than the yield moment because the web and flanges partially plastify before buckling. Appendix A6 is an optional alternative that captures this extra capacity for straight I-sections in negative flexure (and noncomposite sections) when the web is compact or noncompact, the yield strengths do not exceed 70 ksi, and the flange ratio I_{yc}/I_{yt} is at least 0.3, along with other conditions in the appendix.

Instead of flange stresses, A6 works in moments. It uses web plastification factors R_{pc} and R_{pt} to define resistances between M_{yc} and M_p , and then applies flange local buckling and lateral-torsional buckling reductions in moment form. The designer checks $M_u + (1/3)f_l S_{xc} \leq \phi_f M_{nc}$ for the compression flange and a similar equation for the tension flange.

Many modern plate girders have slender webs at the piers ($2D_c/t_w$ greater than λ_{rw}), and for those the appendix does not apply, so Article 6.10.8 governs. For rolled beams and stockier plate girders, Appendix A6 can give a noticeably higher resistance. Appendix B6 is a separate option that allows limited redistribution of negative moment at interior piers for sections that meet its requirements. Both appendices are tools for economy, not requirements; owners may restrict their use, so check the design manual before relying on them.

Whichever route is taken, the same constructibility, service and fatigue checks still apply, because those limit states depend on elastic behavior rather than on the ultimate resistance at strength.

Key takeaways

- Check the 6.10.2 proportion limits first: $D/t_w \leq 150$ unstiffened, $b_f/2t_f \leq 12$, $b_f \geq D/6$, $t_f \geq 1.1t_w$, $0.1 \leq I_{yc}/I_{yt} \leq 10$.
- Unshored elastic stresses sum loads on the steel section, the $3n$ long-term section and the n short-term section.
- A composite section in positive flexure is compact if $F_y \leq 70$ ksi, $D/t_w \leq 150$ and $2D_{cp}/t_w \leq 3.76\sqrt{(E/F_{yc})}$.
- Compact positive-flexure resistance is M_p when $D_p \leq 0.1D_t$, otherwise $M_p(1.07 - 0.7D_p/D_t)$, with a $1.3R_{hm_y}$ cap in some continuous spans.
- Every composite section in positive flexure must meet the ductility limit $D_p \leq 0.42D_t$.
- Discretely braced compression flanges are checked for flange local buckling and lateral-torsional buckling; the smaller F_{nc} governs.
- Lateral-torsional buckling resistance depends on the unbraced length L_b compared with L_p and L_r , and on C_b .
- Constructibility checks the noncomposite steel girder under wet concrete, often controlling top-flange size and cross-frame spacing.

References

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3. Caltrans, Bridge Design Practice, Chapter 6.1 Steel Design Theory, October 2022 (dot.ca.gov)
4. Illinois DOT, Bridge Manual Design Guide 3.3.4, LRFD Composite Steel Beam Design (idot.illinois.gov)
5. FHWA, Steel Bridge Design Handbook, FHWA-HIF-16-002, Vol. 12: Design for Constructibility and Vol. 11: Design Example (2015)

Study text only; the final quiz is taken online. Verify all references against the edition your agency has adopted.