

Live Load Distribution Factors for Beam-Slab Bridges

Once the per-lane HL-93 effects are known (BR-D02), the designer has to decide how much of each lane a single girder carries. For common beam-slab bridges, the AASHTO LRFD Bridge Design Specifications give approximate live load distribution factors in Article 4.6.2.2. This course explains where the equations came from, the conditions for using them, the moment and shear equations for concrete decks on steel or concrete beams, the exterior girder rules (lever rule, e-factor and rigid-section check), the skew corrections, the handling of multiple presence and fatigue, and the signs that a refined analysis is needed. Equations and the worked example were checked against the FHWA Comprehensive Design Example for a Prestressed Concrete Girder Superstructure Bridge (Design Step 5.1) and University of Maryland course notes that reproduce the tables. A second example is an illustrative teaching problem with numbers computed for this course. Always check the edition and interim revisions your owner has adopted.

Learning objectives

- Explain approximate distribution methods
- Compute moment and shear distribution factors
- Apply exterior girder and skew adjustments
- Know when refined analysis is needed

What a distribution factor is

A girder distribution factor, g , is the fraction of one design lane's live load effect that a single girder carries. If the HL-93 moment per lane (with IM) is M_{lane} , the live load moment for an interior girder is $g \times M_{\text{lane}}$. LRFD expresses g in **lanes**. The FHWA design example points out the contrast with the Standard Specifications, which expressed distribution in **wheel lines** (for example the familiar $S/5.5$ for concrete decks on steel or prestressed girders with two or more lanes). One lane has two wheel lines, so a Standard Specifications factor divided by two is roughly comparable to an LRFD factor, but the two methods are not interchangeable because the LRFD equations depend on more variables.

Exact distribution depends on the relative stiffness of the girders and deck, girder spacing, span length, skew, diaphragms and the number of girders. The Standard Specifications' S/D approach captured only spacing. The LRFD approximate equations were developed largely from work in NCHRP Project 12-26, which, as the FHWA example notes, was verified against three-dimensional bridge analyses and field measurements. The resulting equations add span length and a longitudinal stiffness parameter, and they come with ranges of applicability that reflect the range of bridges studied.

Two features of the equations surprise engineers who are new to them. First, **multiple presence is already built in**: the one-lane equation includes the 1.20 factor and the multi-lane equation includes the appropriate factors, so you do not multiply by m again. Second, the factors are applied to the **whole HL-93 lane effect**, truck or tandem with IM plus the lane load, not to individual axles.

Conditions for using the approximate method

Article 4.6.2.2.1 lists the conditions under which the approximate equations apply, unless otherwise stated in a particular table:

- the width of deck is constant;
- unless otherwise specified, the number of beams is not less than four;
- the beams are parallel and have approximately the same stiffness;
- unless otherwise specified, the roadway part of the overhang, d_e , does not exceed 3.0 ft;
- curvature in plan is less than the limit specified in Article 4.6.1.2.4; and
- the cross-section is consistent with one of the cross-sections shown in Table 4.6.2.2.1-1.

The cross-section types in Table 4.6.2.2.1-1 are identified by letters. This course focuses on the most common group: a concrete deck (including filled or partially filled grid decks composite with a reinforced slab) on steel or concrete beams, and concrete T-beams, cross-sections **(a)**, **(e)** and **(k)**, along with (i) and (j) where sufficiently connected to act as a unit. Precast I and bulb-tee girders, for example, are cross-section (k), as the FHWA example notes. Box beams, slab spans, multi-cell boxes and wood decks have their own equations.

Each equation also has a **range of applicability**. For the moment equations for types (a), (e) and (k): girder spacing $3.5 \text{ ft} \leq S \leq 16.0 \text{ ft}$, slab thickness $4.5 \text{ in.} \leq t_s \leq 12.0 \text{ in.}$, span $20 \text{ ft} \leq L \leq 240 \text{ ft}$, number of beams $N_b \geq 4$, and stiffness $10,000 \leq K_g \leq 7,000,000 \text{ in.}^4$. The FHWA example explains what to do outside the ranges: where girder spacing exceeds the listed range, the specifications require the lever rule; where other parameters are only slightly out of range the equations may still be reasonable, but where they greatly exceed the range, engineering judgment is needed, and that usually means a refined analysis.

Interior girders: the moment equations

For concrete decks on steel or concrete beams and concrete T-beams (types a, e and k, and i and j where sufficiently connected to act as a unit), Table 4.6.2.2.2b-1 gives the live load moment distribution per lane for interior beams:

One design lane loaded: $g = 0.06 + (S/14)^{0.4} (S/L)^{0.3} (K_g/12.0L t_s^3)^{0.1}$

Two or more design lanes loaded: $g = 0.075 + (S/9.5)^{0.6} (S/L)^{0.2} (K_g/12.0L t_s^3)^{0.1}$

where S is girder spacing (ft), L is span length (ft), t_s is the depth of the concrete slab (in.) and K_g is the longitudinal stiffness parameter (in.⁴). The governing interior factor is the larger of the one-lane and multi-lane values.

The stiffness parameter

Article 4.6.2.2.1 defines

$$K_g = n (I + A e_g^2)$$

where $n = E_B/E_D$ is the modular ratio of beam to deck, I and A are the moment of inertia and area of the noncomposite beam, and e_g is the distance between the centers of gravity of the basic beam and the deck. In the FHWA prestressed girder example, e_g is taken as the distance from the beam centroid to the top of the beam plus half the slab thickness, ignoring the haunch; the example notes that ignoring the integral wearing surface would make only a minimal difference.

Notice the exponent of 0.1 on the stiffness term. Even a large change in K_g changes g only modestly: doubling K_g multiplies the term by $2^{0.1} \approx 1.07$. That is why preliminary design often proceeds with an estimated stiffness term, and why the factor

rarely needs to be recomputed when a girder size changes slightly. Article 4.6.2.2.1 permits simplified values of the stiffness term for preliminary design, given in Table 4.6.2.2.1-2; use the computed value for final design.

What the exponents tell you

- **Spacing (S).** The strongest variable. Wider spacing means each girder carries more of each lane.
- **Span (S/L).** Longer spans distribute load better between girders, so g falls as L grows for a given S . The Standard Specifications S/5.5 did not capture this, which is one reason LRFD factors are often lower than older factors on long spans and higher on short spans.
- **Stiffness.** Stiffer girders relative to the slab carry more load locally; the slab is less able to spread it.

Continuous spans

For continuous girders, the span length used in the equations depends on the force effect. Article 4.6.2.2.1 refers to Table C4.6.2.2.1-1 (in the commentary) for the length L to use, for example the length of the span under consideration for positive moment, and the average of the two adjacent spans for negative moment near an interior support. Owners frequently set this in their design manuals; check yours. The stiffness term is typically computed with section properties from the region being designed.

Three-girder bridges

The equations assume at least four beams. Table 4.6.2.2.2b-1 addresses $N_b = 3$ for these cross-section types: use the lesser of the value from the equation and the lever rule. Narrow bridges with three girders are where many approximate-method errors occur, because the lever rule and the equation can give quite different results.

Interior girders: the shear equations

Table 4.6.2.2.3a-1 gives the live load shear distribution per lane for interior beams of types a, e and k (and i and j if sufficiently connected):

One design lane loaded: $g = 0.36 + S/25.0$

Two or more design lanes loaded: $g = 0.2 + S/12 - (S/35)^{2.0}$

The range of applicability is $3.5 \text{ ft} \leq S \leq 16.0 \text{ ft}$, $20 \text{ ft} \leq L \leq 240 \text{ ft}$, $4.5 \text{ in.} \leq t_s \leq 12.0 \text{ in.}$ and $N_b \geq 4$, as restated in the University of Maryland course notes; as with moment, a three-girder bridge uses the lever rule. The shear equations depend only on spacing. Shear near a support is dominated by the wheel loads closest to the girder, and the research behind the equations found span and stiffness had less influence than for moment.

Comparing the moment and shear factors for the same girder is a useful sanity check. Shear factors are normally higher than moment factors, because the critical shear position puts a heavy axle right next to the support where the deck has little length over which to spread the load. In the FHWA prestressed example ($S = 9.667 \text{ ft}$), the interior moment factor was 0.796 lane and the interior shear factor before skew correction was 0.929 lane.

Using the factors correctly

- Apply the factor to the full lane effect: (truck or tandem) $\times (1 + \text{IM})$ + lane load, from BR-D02. Do not apply IM after distribution in a way that double counts, and do not apply it to the lane load.

- Use the moment factor for moments, including negative moments over piers (with the appropriate L), and the shear factor for shears and reactions.
- The shear factor applies to support shear and to shear along the span in practice; owners differ on whether the obtuse-corner skew correction is applied along the whole span or only near the support (discussed later).
- Do not apply multiple presence again. The equations include it.

Standard Specifications comparison

Under the Standard Specifications, a two-lane concrete deck on steel or prestressed girders used S/5.5 wheel lines for moment, about S/11 lanes. For S = 8 ft that is about 0.73 lane. The LRFD multi-lane interior moment factor for an 8-ft spacing ranges with span; for the illustrative 80-ft steel girder problem later in this course it is about 0.69 lane. The values are similar here, but because the LRFD equations also respond to span length and stiffness, the two methods can diverge noticeably for short or long spans. That is one reason rating an older bridge to LRFR can give different results from its original design calculations even before any deterioration.

Exterior girders: lever rule, e-factor and rigid section

Exterior girders get three separate checks, and the governing factor is the largest that applies.

1. One lane loaded: the lever rule

For one design lane loaded on an exterior girder of types a, e and k, Tables 4.6.2.2.2d-1 (moment) and 4.6.2.2.3b-1 (shear) call for the **lever rule**. The commentary describes the method: the deck is assumed to be hinged over the first interior girder, and the reaction at the exterior girder is found by statics, summing moments about the hinge. The truck's wheel lines are 6.0 ft apart and the outer wheel is placed 2.0 ft from the face of the barrier (the lane-edge limit for components other than the overhang, applied at the barrier because the design lane starts there). Any wheel landing inboard of the first interior girder contributes nothing to the exterior reaction.

Because the lever rule is a statical calculation, **it does not include multiple presence**. For strength and service, multiply by $m = 1.20$. For fatigue, use the value without m . The FHWA prestressed example shows this explicitly: the lever rule gave 0.672 lane, used directly for fatigue, and $1.20 \times 0.672 = 0.806$ lane for strength and service, which governed the exterior girder moment.

2. Two or more lanes: the e-factor

For two or more lanes, the exterior girder factor is a correction to the interior factor:

Moment: $g_{\text{ext}} = e \times g_{\text{int}}$, with $e = 0.77 + d_e/9.1$

Shear: $g_{\text{ext}} = e \times g_{\text{int}}$, with $e = 0.6 + d_e/10$

where d_e is the horizontal distance in feet from the exterior web of the exterior beam to the interior edge of the curb or traffic barrier, positive when the web is inboard of the barrier, with a range of applicability of $-1.0 \text{ ft} \leq d_e \leq 5.5 \text{ ft}$. In the FHWA example, $d_e = 1.83 \text{ ft}$ gave $e = 0.97$ for moment and 0.783 for shear.

The overhang matters a great deal. A long roadway overhang (large d_e) puts wheels outboard of the exterior girder and increases its share; that is why Article 4.6.2.2.1 lists $d_e \leq 3.0 \text{ ft}$ as a general condition for the approximate method, and why

the specification's range on the e-factor equation is limited.

3. Rigid-section check with diaphragms or cross-frames

Article 4.6.2.2.2d requires that, in beam-slab bridge cross-sections with diaphragms or cross-frames, the distribution factor for the exterior beam not be taken less than that which would be obtained by assuming the cross-section deflects and rotates as a rigid cross-section. The FHWA example explains why: the original study that developed the equations did not consider intermediate diaphragms. The commentary gives the familiar pile-group style equation:

$$R = N_L / N_b + X_{\text{ext}} \Sigma e / \Sigma x^2$$

where R is the reaction on the exterior beam in lanes, N_L the number of loaded lanes, N_b the number of beams, e the eccentricity of each design truck or lane load from the center of gravity of the girder pattern, x the distance from that center of gravity to each girder, and X_{ext} the distance to the exterior girder. Multiple presence is applied to the result for each number of loaded lanes. Note that eccentricities of lanes on the far side of the center are negative.

Whether the rigid-section check applies depends on the diaphragms. The FHWA example notes that application requires enough intermediate diaphragms of adequate stiffness to force the cross-section to act rigidly, and that practice for prestressed girders varies by jurisdiction. Steel I-girder bridges with full-depth cross-frames are the usual case where it applies. In the FHWA example, with one deep concrete diaphragm at midspan judged sufficient, the rigid-section results were 0.572 (one lane, with m), 0.776 (two lanes) and 0.764 (three lanes), none of which controlled over the 0.806 lever rule value for moment.

Skewed bridges

Skew changes the way load travels to the supports. In a skewed beam-slab bridge, the deck and cross-frames carry part of the load across the skew toward the obtuse corners, where the supports are closest together along a line perpendicular to the girders. The result is that midspan moments in the girders tend to be lower than for a right bridge of the same span, and support shears and reactions at the obtuse corners tend to be higher. The LRFD approximate method captures both effects with correction factors.

Moment reduction (Article 4.6.2.2.2e)

For types a, e and k (and i and j where connected), Table 4.6.2.2.2e-1 permits the moment distribution factor to be reduced by

$$1 - c_1 (\tan \theta)^{1.5}, \quad c_1 = 0.25 (K_g / 12.0 L t_s^3)^{0.25} (S/L)^{0.5}$$

where θ is the skew angle. If θ is less than 30° , $c_1 = 0$, so no reduction is taken. If θ exceeds 60° , θ is taken as 60° . The range of applicability is $30^\circ \leq \theta \leq 60^\circ$, with the same S, L and N_b limits as the moment equations. The FHWA prestressed example, with a 20° skew, therefore applied no moment reduction.

The reduction is permissive ("may be reduced"). Some owners do not allow it, or allow it only for interior girders, because skewed bridges have other complexities, such as torsion, cross-frame forces and uplift at acute corners, that a reduced moment factor does not address. Check the owner's policy.

Shear correction at the obtuse corner (Article 4.6.2.2.3c)

For support shear at the obtuse corner, Table 4.6.2.2.3c-1 gives a correction factor for the same cross-section types:

$$1.0 + 0.20 (12.0L t_s^3 / K_g)^{0.3} \tan \theta$$

with a range of $0^\circ \leq \theta \leq 60^\circ$, $3.5 \text{ ft} \leq S \leq 16.0 \text{ ft}$, $20 \text{ ft} \leq L \leq 240 \text{ ft}$ and $N_b \geq 4$. Unlike the moment reduction, this is an **increase**, and it applies to all skews, not just those over 30° . In the FHWA example, a 20° skew gave a correction of 1.047, raising the interior two-lane shear factor from 0.929 to 0.973 lane and the exterior one-lane factor from 0.806 to 0.845 lane.

The University of Maryland notes, summarizing the commentary, point out that the correction targets the girder at the obtuse corner, that it is valid for skew angles up to 60° , and that it may be conservatively applied to all end shears. Many owners simply apply it to every girder at every support, which is conservative and simple. Others apply it only to the exterior girder at the obtuse corner and to the interior girders as directed in the commentary. The correction is applied to the shear near the support; how far into the span it is carried is a matter of owner policy, so record the choice in the design criteria.

Sanity check with numbers

In the illustrative steel problem later in this course ($S = 8 \text{ ft}$, $L = 80 \text{ ft}$, $t_s = 8.5 \text{ in.}$, $K_g = 1,200,000 \text{ in.}^4$), the stiffness ratio $K_g / 12L t_s^3$ is about 2.04. At a 40° skew (an assumed value for teaching), $c_1 \approx 0.094$ and the moment reduction factor is about 0.927, while the obtuse-corner shear correction is about 1.136. So, for this example, skew reduces the interior moment factor by about 7 percent and increases the support shear factor by about 14 percent. Such numbers explain why skewed bridges often have shear and bearing designs governed at the obtuse corner.

When skew is too much for the approximate method

Large skews, particularly combined with short spans, wide decks, staggered cross-frames or variable skew between supports, produce behavior the equations were not developed to capture: twisting of girders, large cross-frame forces, uplift at acute corners and significant differences in girder deflection during deck placement. The 60° cap on θ is a clue that the equations were not intended for extreme skews. Such bridges are good candidates for refined analysis, as discussed later in this course.

Source-based example: FHWA prestressed girder bridge

The FHWA Comprehensive Design Example for a Prestressed Concrete Girder Superstructure Bridge walks through distribution factors in Design Step 5.1. It is worth reproducing the logic because it covers every rule in this course. The example was prepared with an earlier edition of the LRFD specifications; the equations used here are the same forms reproduced in current-edition references, but verify against your adopted edition.

Inputs (from the FHWA example)

- AASHTO Type I-Beam (28/72): $A = 1,085 \text{ in.}^2$, $I = 733,320 \text{ in.}^4$, centroid 35.62 in. below the top of the beam
- Slab $t_s = 8 \text{ in.}$; span $L = 110 \text{ ft}$; spacing $S = 9 \text{ ft } 8 \text{ in. (9.667 ft)}$; six girders; skew 20°
- $E_B = 4,696 \text{ ksi}$, $E_D = 3,834 \text{ ksi}$, so $n = 1.225$
- $d_e = 1.83 \text{ ft}$

Stiffness

$$e_g = 35.62 + 8/2 = 39.62 \text{ in.}; K_g = 1.225 [733,320 + 1,085 (39.62)^2] = 2,984,704 \text{ in.}^4.$$

Interior girder

- Moment, two or more lanes: 0.796 lane. One lane: 0.542 lane. Governing: 0.796.
- Moment skew reduction: not applied ($20^\circ < 30^\circ$).
- Shear, two or more lanes: $0.2 + 9.667/12 - (9.667/35)^2 = 0.929$; $\times 1.047$ skew correction = 0.973 lane.
- Shear, one lane: $0.36 + 9.667/25 = 0.747$; $\times 1.047 = 0.782$ lane.
- Fatigue: moment $0.542/1.2 = 0.452$ lane; shear $0.782/1.2 = 0.652$ lane.

Reproducing these numbers yourself is an excellent way to verify a spreadsheet: with the inputs above, the moment equations give 0.542 and 0.796 to three decimals.

Exterior girder

- Moment, two or more lanes: $e = 0.77 + 1.83/9.1 = 0.97$; $0.97 \times 0.796 = 0.772$ lane.
- Moment, one lane, lever rule: 0.672 lane without m (fatigue); $1.2 \times 0.672 = 0.806$ lane (strength and service).
- Shear, two or more lanes: $e = 0.6 + 1.83/10 = 0.783$; $0.783 \times 0.973 = 0.762$ lane.
- Shear, one lane, lever rule with skew: $1.047 \times 0.672 = 0.704$ (fatigue); $1.047 \times 0.806 = 0.845$ (strength and service).
- Rigid-section check (one deep midspan diaphragm judged sufficient): 0.572, 0.776 and 0.764 for one, two and three lanes with m. None controlled.

Results

	Interior moment	Exterior moment	Interior shear	Exterior shear
Strength / service design value	0.796	0.806	0.973	0.845
Fatigue design value	0.452	0.672	0.652	0.704

Observations: the exterior girder moment was governed by the one-lane lever rule with $m = 1.20$, not by the multi-lane equation; the shear factors were higher than the moment factors; and the skew correction affected only shear. The FHWA example also notes that fatigue of the prestressing steel need not be checked for conventional prestressed girders meeting the Service III stress limits, but computes fatigue factors anyway for completeness, which is why they are shown.

Illustrative example: five-girder steel bridge

This problem uses assumed dimensions chosen for teaching. The numbers were computed for this course and are not from a real design. They show the full sequence for a steel I-girder bridge with cross-frames.

Assumed data

- Simple span $L = 80$ ft; five steel girders at $S = 8.0$ ft; slab $t_s = 8.5$ in.
- $K_g = 1,200,000 \text{ in.}^4$ (assumed; in practice computed as $n(I + Ae_g^2)$ with the steel-to-concrete modular ratio)
- Barrier inside faces 2.0 ft outboard of the exterior girder webs ($d_e = 2.0$ ft), so the clear roadway is $4 \times 8 + 2 \times 2 = 36$ ft: three design lanes.
- Full-depth cross-frames, so the rigid-section check applies.
- Skew taken as 0° for the main example; a 40° variation is shown at the end.

Check applicability

$S = 8.0$ (3.5 to 16.0, OK); $t_s = 8.5$ (4.5 to 12.0, OK); $L = 80$ (20 to 240, OK); $N_b = 5$ (≥ 4 , OK); $K_g = 1.2 \times 10^6$ (10,000 to 7,000,000, OK); $d_e = 2.0$ (≤ 3.0 general limit and within -1.0 to 5.5 , OK).

Interior girder

Stiffness term: $K_g/(12 \times 80 \times 8.5^3) = 1,200,000/589,560 \approx 2.04$.

- Moment, one lane: $0.06 + (8/14)^{0.4}(8/80)^{0.3}(2.04)^{0.1} \approx 0.490$
- Moment, two or more lanes: $0.075 + (8/9.5)^{0.6}(8/80)^{0.2}(2.04)^{0.1} \approx 0.686 \rightarrow$ governs
- Shear, one lane: $0.36 + 8/25 = 0.680$
- Shear, two or more lanes: $0.2 + 8/12 - (8/35)^2 \approx 0.814 \rightarrow$ governs

Exterior girder

- **Lever rule, one lane.** The outer wheel is 2.0 ft from the barrier face, which is directly over the exterior girder ($d_e = 2.0$ ft). The inner wheel is 6.0 ft inboard, 2.0 ft from the first interior girder. Summing moments about the first interior girder: $R = [0.5 \times 8 + 0.5 \times 2]/8 = 0.625$ lane. With $m = 1.20$: 0.750 lane (strength and service); 0.625 lane for fatigue.
- **e-factor, two or more lanes.** Moment: $e = 0.77 + 2.0/9.1 \approx 0.990$; $g \approx 0.990 \times 0.686 \approx 0.679$. Shear: $e = 0.6 + 2.0/10 = 0.80$; $g \approx 0.80 \times 0.814 \approx 0.651$.
- **Rigid section.** Girders are at $x = \pm 16, \pm 8$ and 0 ft from the center, so $\Sigma x^2 = 2(16^2 + 8^2) = 640 \text{ ft}^2$ and $X_{\text{ext}} = 16$ ft. Taking each truck's center 5 ft inside its lane edge (2 ft edge distance plus half the 6-ft gage), with the first lane against the barrier at 18 ft from center, the truck eccentricities are 13, 1 and -11 ft. One lane: $R = 1/5 + 16(13)/640 = 0.525$; $\times 1.20 = 0.630$. Two lanes: $R = 2/5 + 16(14)/640 = 0.750$; $\times 1.00 = 0.750$. Three lanes: $R = 3/5 + 16(3)/640 = 0.675$; $\times 0.85 \approx 0.574$.

The exterior moment factor is the largest of 0.750 (lever rule), 0.679 (e-factor) and 0.750 (rigid section, two lanes): 0.750 lane. The exterior shear factor is the largest of 0.750 (lever rule) and 0.651 (e-factor), with the rigid-section value of 0.750 also considered where the owner applies it to shear: 0.750 lane.

Fatigue

Interior moment: $0.490/1.2 \approx 0.408$ lane (one-lane equation with m removed). Exterior moment: the larger of the lever rule without m (0.625) and the one-lane rigid-section value without m (0.525): 0.625 lane.

Live load moment per girder

Using the illustrative BR-D02 style simple-span numbers for 80 ft, the reader can compute the per-lane LL + IM moment and multiply by 0.686 (interior) or 0.750 (exterior). The point of the example is the sequence, not the final moment.

Variation: 40° skew (assumed)

$c_1 = 0.25(2.04)^{0.25}(0.1)^{0.5} \approx 0.094$; moment reduction $1 - 0.094(\tan 40^\circ)^{1.5} \approx 0.927$, so the interior moment factor would become about 0.636 if the owner permits the reduction. Obtuse-corner shear correction: $1 + 0.20(1/2.04)^{0.3} \tan 40^\circ \approx 1.136$, so the interior shear factor would become about 0.925 lane at the obtuse corner.

When refined analysis is needed

Article 4.6.2.2 is an approximate method. Article 4.6.3 permits refined methods, such as grillage (grid) analysis, plate and finite strip methods, and finite element analysis, for any bridge, and the FHWA example notes at the outset that the specifications allow advanced methods of analysis to determine distribution. The question is when the approximate method stops being good enough.

Clear triggers

- **Outside the conditions of Article 4.6.2.2.1.** Variable deck width, non-parallel girders, girders of very different stiffness, roadway overhang beyond 3.0 ft, or a cross-section that does not match Table 4.6.2.2.1-1.
- **Horizontally curved girders** beyond the limits of Article 4.6.1.2.4. Curved steel I-girder and box girder bridges usually need refined analysis because girder forces depend on the curvature and the cross-frame system, not just spacing.
- **Parameters far outside the ranges of applicability.** Spacing beyond the range sends you to the lever rule by specification; other parameters far out of range call for judgment, often refined analysis.
- **Severe or variable skew.** Skews near or beyond 60°, skew that changes between supports, or skewed bridges with staggered or discontinuous cross-frames.
- **Unusual framing.** Floorbeam and stringer systems, integral pier caps, girders that frame into other girders, or bridges with large openings in the deck.

Cases where refined analysis often pays

- **Load rating.** When a bridge rates below 1.0 for a legal load using approximate factors, a refined analysis can show more favorable distribution. Many owners accept refined analysis for rating, sometimes with conditions; this ties into the load rating courses (BR-L01 to BR-L06).
- **Wide bridges with many girders,** where the approximate multi-lane factor may be conservative for interior girders.
- **Widenings and staged construction,** where old and new girders differ in stiffness and the approximate method's "approximately the same stiffness" condition is not met.

Doing refined analysis well

- **Place lanes and vehicles explicitly.** A refined model does not include multiple presence. The analyst loads one lane with $m = 1.20$, two lanes with 1.00, three with 0.85 and more with 0.65, moving lanes across the deck within the edge limits (BR-D02), and keeps the extreme result for each girder and effect.
- **Model the elements that drive distribution:** deck stiffness, girder composite properties, cross-frames or diaphragms and, where relevant, barriers. Some owners do not allow barrier stiffness to be counted for strength; check policy.
- **Watch the exterior girder rule.** Article 4.6.2.2.2d's rigid-section minimum is part of the approximate method; owners differ on whether a minimum is applied to refined results. Some owners require that refined distribution factors not be less than a fraction of the approximate factors.
- **Verify.** Check the total live load in the model against the applied loads, compare girder results with the approximate factors as a reasonableness check, and investigate any large differences.

Research since the original equations

The equations in Article 4.6.2.2 were developed from NCHRP Project 12-26. NCHRP Report 592, Simplified Live Load Distribution Factor Equations, examined simpler alternatives based on the lever rule with calibrated adjustments, and

compared them with refined analyses of a large set of bridges. Engineers interested in why the equations look the way they do, or how much scatter there is between approximate and refined results, will find that report useful. The equations in the specification itself remain those in Article 4.6.2.2 of the adopted edition.

Checklist, common errors and summary

A step-by-step checklist

1. **Identify the cross-section type** in Table 4.6.2.2.1-1 (for example a, e or k for concrete decks on steel or concrete beams).
2. **Check the general conditions** of Article 4.6.2.2.1: constant width, at least four beams (or the three-beam rule), parallel beams of similar stiffness, $d_e \leq 3.0$ ft, curvature within limits.
3. **Check each equation's range of applicability**: S , t_s , L , N_b , K_g , d_e and θ .
4. **Compute $K_g = n(I + Ae_g^2)$** with the right modular ratio and e_g ; choose the span length L for each effect per the owner's practice.
5. **Interior girder**: one-lane and multi-lane moment and shear; take the larger of each.
6. **Exterior girder**: lever rule with $m = 1.20$ for one lane; e-factor times interior for two or more lanes; rigid-section check with m where diaphragms or cross-frames are present; take the largest.
7. **Skew**: apply the moment reduction for 30° to 60° only if permitted; apply the obtuse-corner shear correction per owner policy.
8. **Fatigue**: one-lane equation values divided by 1.20; lever-rule and rigid-section values without m .
9. **Apply** the factors to the full per-lane effect from BR-D02, (truck or tandem)(1 + IM) + lane, and then apply load factors (BR-D01).
10. **Document** every choice in the design criteria, and hand check at least one interior and one exterior girder.

Common errors

- Multiplying the equation results by multiple presence factors (double counting).
- Forgetting $m = 1.20$ on the lever rule for strength and service.
- Using the one-lane equation value directly for fatigue without dividing by 1.20.
- Measuring d_e to the centerline of the girder instead of the exterior web, or to the outside of the barrier instead of its inside face. (Check the definition in the adopted edition; the difference is half a web thickness, small for prestressed girders, but sign errors on d_e are not small.)
- Skipping the rigid-section check on a steel girder bridge with cross-frames.
- Applying the moment skew reduction for skews under 30° , or extrapolating beyond 60° .
- Using the approximate method for a curved, flared or widened bridge outside the conditions.
- Using moment factors for shear or reactions.

Self-check questions

- Why does a single-lane distribution factor from the equations need to be divided by 1.20 for fatigue, while a lever-rule value without m does not?
- For the FHWA example, why did the exterior girder moment factor come from the lever rule instead of the e-factor equation?
- Your bridge has three girders. What do you do?

- Your bridge has a 25° skew. Which correction applies, and which does not?
- Your steel girder bridge has full-depth cross-frames. Which additional exterior girder check is required?

Summary

Distribution factors convert per-lane HL-93 effects into girder effects. For common beam-slab bridges, Article 4.6.2.2 provides calibrated equations that include multiple presence and depend on spacing, span and stiffness for moment and on spacing alone for shear. Interior girders use the larger of one-lane and multi-lane results. Exterior girders use the largest of the lever rule (with m), the e -factor adjustment and, where diaphragms or cross-frames act, the rigid-section check. Skew permits a moment reduction for 30° to 60° and requires a shear increase at the obtuse corner. Fatigue needs the 1.20 removed. When the bridge falls outside the conditions or ranges, or its behavior is clearly three-dimensional, a refined analysis with explicit lane placement and multiple presence is the right tool.

Beyond I-girders: other superstructures and load rating

The equations covered so far apply to concrete decks on steel or concrete beams and similar T-sections. Other superstructure types in Table 4.6.2.2.1-1 have their own provisions, and the same distribution concepts carry into load rating.

Other beam-slab and multi-beam types

- **Concrete deck on spread box beams** (types b and c) and **cast-in-place multicell boxes** (type d) have separate moment and shear equations with their own variables, such as beam depth d and number of cells, and their own skew corrections. The University of Maryland notes reproduce the shear correction factors for these types alongside the I-girder version, which is a reminder that the obtuse-corner correction is not one equation for every bridge.
- **Adjacent box and slab beams used in multi-beam decks** (types f and g) use equations that include beam width, depth and torsional stiffness, and they apply only if the beams are *sufficiently connected to act as a unit*, typically by transverse post-tensioning or a structural deck. Adjacent beams tied only by shear keys that have cracked may not meet that condition, which is common in older bridges and matters in rating.
- **Wood decks and concrete decks on wood beams** use their own tables, which for some cases simply call for the lever rule.

Slab bridges: equivalent strip widths

Slab-type bridges are not distributed with girder factors but with an **equivalent strip width** in Article 4.6.2.3. The width of strip carrying one lane is $E = 10.0 + 5.0(L_1 W_1)^{0.5}$ in inches for one lane loaded, and $E = 84.0 + 1.44(L_1 W_1)^{0.5} \leq 12.0W/N_L$ in inches for more than one lane loaded, where L_1 is the modified span length and W_1 the modified edge-to-edge width in feet, each limited as specified in that article, W is the physical edge-to-edge width and N_L is the number of design lanes. As with the girder equations, multiple presence is accounted for in the formulation. Skewed slab bridges get a reduction of longitudinal force effects in the same article.

Distribution in LRFR load rating

The AASHTO Manual for Bridge Evaluation uses the LRFD distribution provisions for LRFR ratings. For the HL-93 design load rating and for legal load ratings, the same approximate factors (or a refined analysis) are used as in design. Permit ratings introduce a nuance: for some permit categories, such as special or escorted single-trip permits where the permit vehicle is expected to cross with no other heavy vehicles alongside, the manual directs use of a one-lane distribution factor with the

built-in multiple presence factor removed. That is the same "divide by 1.20" operation used for fatigue, for a different reason. Check the edition of the manual and the owner's rating policy before applying it.

Distribution factors are often where rating results gain or lose the most. An older bridge designed with S/5.5 may rate differently with LRFD factors; a refined analysis may recover capacity on a wide bridge; and an adjacent box beam bridge with failed shear keys may need the lever rule or a conservative assumption because the "acts as a unit" condition is no longer met. Field condition, not just the plans, can decide which distribution method is defensible.

Deck design is different

Finally, remember that the deck itself is designed with a different method. The approximate strip method of Article 4.6.2.1 distributes wheel loads into transverse or longitudinal strips of deck with widths that depend on the strip direction and the sign of moment, and the empirical deck design method of Section 9 is another option where permitted. Girder distribution factors are for the girders only.

Quick reference: types a, e and k

The table below collects the equations from this course for concrete decks on steel or concrete beams and concrete T-beams. It is a study aid. Design from the adopted edition of the specification, including its notes and ranges.

Effect	One lane loaded	Two or more lanes loaded
Interior moment	$0.06 + (S/14)^{0.4}(S/L)^{0.3}(K_g/12L_t^3)^{0.1}$	$0.075 + (S/9.5)^{0.6}(S/L)^{0.2}(K_g/12L_t^3)^{0.1}$
Interior shear	$0.36 + S/25$	$0.2 + S/12 - (S/35)^2$
Exterior moment	Lever rule $\times 1.20$	$e \times \text{interior}$, $e = 0.77 + d_e/9.1$
Exterior shear	Lever rule $\times 1.20$	$e \times \text{interior}$, $e = 0.6 + d_e/10$
Exterior, with cross-frames or diaphragms	Not less than rigid section: $R = N_L/N_b + X_{\text{ext}}\Sigma e/\Sigma x^2$, times m	
Skew, moment	$\times [1 - c_1(\tan \theta)^{1.5}]$, $c_1 = 0.25(K_g/12L_t^3)^{0.25}(S/L)^{0.5}$; $c_1 = 0$ below 30° ; $\theta \leq 60^\circ$	
Skew, support shear (obtuse corner)	$\times [1.0 + 0.20(12L_t^3/K_g)^{0.3} \tan \theta]$, $0^\circ \leq \theta \leq 60^\circ$	
Fatigue	One-lane equation value $\div 1.20$; lever rule and rigid section without m	

Ranges for the I-girder equations: $3.5 \leq S \leq 16.0$ ft; $4.5 \leq t_s \leq 12.0$ in.; $20 \leq L \leq 240$ ft; $N_b \geq 4$ (three-girder rule applies otherwise); $10,000 \leq K_g \leq 7,000,000$ in.⁴ for moment; $-1.0 \leq d_e \leq 5.5$ ft for the e-factors. Units: S, L and d_e in feet; t_s in inches; K_g in in.⁴.

Key takeaways

- A distribution factor g is the fraction of one design lane's HL-93 effect (with IM) carried by a girder, in lanes (LRFD) rather than wheel lines (Standard Specifications).
- The approximate equations of Article 4.6.2.2 came largely from NCHRP 12-26 and already include multiple presence, except where the lever rule is called for.
- Use them only within Article 4.6.2.2.1 conditions and each table's range of applicability; beyond the spacing range, use the lever rule.

- Interior moment, types a/e/k: one lane $0.06 + (S/14)^{0.4}(S/L)^{0.3}(K_g/12L_t^3)^{0.1}$; two or more lanes $0.075 + (S/9.5)^{0.6}(S/L)^{0.2}(K_g/12L_t^3)^{0.1}$.
- Interior shear: one lane $0.36 + S/25$; two or more lanes $0.2 + S/12 - (S/35)^2$.
- Exterior: lever rule for one lane (add $m = 1.20$); $e = 0.77 + d_e/9.1$ for moment and $0.6 + d_e/10$ for shear with two or more lanes; rigid-section check where diaphragms or cross-frames are present.
- Skew: moment may be reduced for 30° to 60° skews; support shear at the obtuse corner is increased by $1 + 0.20(12L_t^3/K_g)^{0.3}\tan\theta$.
- For fatigue, divide single-lane equation values by 1.20; lever-rule values without m can be used directly.

References

1. AASHTO, LRFD Bridge Design Specifications, 9th Edition, 2020
2. Federal Highway Administration, Comprehensive Design Example for Prestressed Concrete (PSC) Girder Superstructure Bridge, Design Step 5.1, Live Load Distribution Factors (fhwa.dot.gov/bridge/lrfd/pscus051.cfm)
3. Fu, C. C., Part 2: The AASHTO LRFD Specifications, course notes, University of Maryland (user.eng.umd.edu/~ccfu/ref/Part2_NEWNEW-2008.pdf)
4. Puckett, J. A., et al., Simplified Live Load Distribution Factor Equations, NCHRP Report 592, Transportation Research Board, 2007

Study text only; the final quiz is taken online. Verify all references against the edition your agency has adopted.